

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2010

Fifth Semester

Computer Science and Engineering

MA 2265 — DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours Maximum : 100 Marks

Answer ALL questions

PART A — (10 × 2 = 20 Marks)

1. When do you say that two compound propositions are equivalent?
2. Prove that  $p \rightarrow q, q \rightarrow r \Rightarrow p \rightarrow r$ .
3. State Pigeonhole principle.
4. Find the recurrence relation satisfying the equation  $n^2 y_n = A(3) + B(4)$ .
5. Define strongly connected graph.
6. State the necessary and sufficient conditions for the existence of an Eulerian path in a connected graph.
7. State any two properties of a group.
8. Define a commutative ring.
9. Define Boolean algebra.
10. Define sub-lattice.

PART B — (5 × 16 = 80 Marks)

11. (a) (i) Prove that the premises  $a \rightarrow (b \rightarrow c)$ ,  $d \rightarrow (b \rightarrow c)$  and  $(a \rightarrow d)$  are inconsistent. (8)  
(ii) Obtain the principal disjunctive normal form and principal conjunctive form of the statement  $p \rightarrow (p \rightarrow (q \rightarrow (q \rightarrow r)))$ . (8)

Or

(b) (i) Prove that

$$\exists x(P(x) \wedge Q(x)), \exists x(R(x) \wedge Q(x)) \Rightarrow \exists x(R(x) \wedge P(x)). \quad (8)$$

(ii) Without using the truth table, prove that

$$p \wedge (q \wedge r) \wedge q \wedge (p \wedge r). \quad (8)$$

12. (a) (i) Prove, by mathematical induction, that for all  $n \geq 1$ ,  $n^2n^3 + 1$  is a multiple of 3. (8)

(ii) Using the generating function, solve the difference equation

$$6y_0 + 2y_1 + y_2 = n + n + n \quad y_0 = 1, \quad y_1 = 2, \quad y_2 = 0. \quad (8)$$

Or

(b) (i) How many positive integers  $n$  can be formed using the digits

3, 4, 4, 5, 5, 6, 7 if  $n$  has to exceed 5000000? (8)

(ii) Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2, 3, 5, 7. (8)

13. (a) (i) Draw the complete graph  $K_5$  with vertices A, B, C, D, E. Draw all complete sub graph of  $K_5$  with 4 vertices. (8)

(ii) If all the vertices of an undirected graph are each of degree  $k$ , show that the number of edges of the graph is a multiple of  $k$ . (8)

Or

(b) (i) Draw the graph with 5 vertices, A, B, C, D, E such that  $\deg(A) = 3$ , B is an odd vertex,  $\deg(C) = 2$  and D and E are adjacent. (8)

(ii) The adjacency matrices of two pairs of graph as given below.

Examine the isomorphism of G and H by finding a permutation matrix.

B.E./B.Tech DEGREE EXAMINATION, MAY/JUNE 2007.

Fifth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1256 - DISCRETE MATHEMATICS

(Common to B.E. (Part-Time) Fourth Semester Regulation 2005)

Time : Three hours Maximum: 100 marks

Answer ALL questions.

PART A - (10 x 2 = 20 marks)

1. Express the statement "Good food is not cheap" in symbolic form.
2. Obtain PDNF for  $\neg(P \vee Q)$ .
3. Define simple statement function.
4. Express the statement "For every 'x' there exist a 'y' such that  $x^2 + y^2 > 100$ " in symbolic form.
5. Give an example of a relation which is symmetric, transitive but not reflexive on  $\{ a, b, c \}$ .
6. Define partially ordered set.
7. If A has 3 elements and B has 2 elements, how many functions are there from A to B.
8. Define Characteristic function.
9. Give an example of sub semi-group.
10. Define normal subgroup of a group.

PART B - (5 x 16 = 80 marks)

11. (a) (i)

Without using truth tables, show that  $Q \vee (P \neg Q) \supset P \neg Q$  is a tautology. Obtain the PDNF for

$(P \supset Q) \wedge (\neg P \supset Q) \supset (Q \vee ?)$ .

Or

Show that S is valid inference from the premises

$P \supset \neg Q, Q \vee R, \neg S \vee P$  and  $\neg R$ .

Without using truth tables, show that

$(\neg P \supset (\neg I \wedge E)) \vee (Q \wedge R) \vee (P \wedge \neg B) \supset \neg R$ .

a) (B)

(8)

(8)

premises

(8)

(b) (i)

12. (a) (i)

(ii)

(ii)

(8)

(8)

(ii)

Show that

$(\forall x)(P(x) \supset \exists y)(Q(y) \wedge R(x,y)) \supset (\forall x)(P(x) \supset \exists y)(R(x,y))$

Is the following conclusion validly derivable from the given?

If  $(\forall x)(P(x) \supset Q(x)), (\exists y)P(y) \supset (\exists z)P(z)$

Or

(b) (i) Verify the validity of the inference. If one person is more successful

than another, then he has worked harder to deserve success.

John has not worked harder than Peter. Therefore. John is not successful than Peter. (8)

13. (a) (i)

Show that  $R \vee S$  is a valid conclusion from the premises

$C \vee D, (C \vee D) \rightarrow H, \neg H \rightarrow (A \rightarrow \neg B)$  and  $(A \rightarrow \neg B) \rightarrow (F \vee S)$

A survey of 500 television watchers produced the following

information: 285 watch foot ball games; 195 watch hockey games:

115 watch Basket ball games: 45 watch foot ball and basket ball

games: 70 watch foot ball and hockey games: 50 watch hockey and

basket ball games: 50 do not watch any of the three games: How

many people watch exactly one of the three games.

(ii) In a Boolean algebra  $Z$  prove that

I

$(a \cap b) = a' \vee b'$  for all  $a, b \in L$ .

(8)

(ii)

(8)

Or

(b) (i)

(ii)

(ii)

Let the relation  $R$  be defined on the set of all real numbers by

'if  $x, y$  are real numbers,  $xRy$  iff  $x - y$  is a rational number'.

Show that  $R$  is an equivalence relation. (8)

Draw the Hasse Diagram of the Lattice  $L$  of all subsets of  $\{a,b,c\}$  under intersection and union, (8)

74. (a) (i)

Or

(i) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 2x - 3$ . Find a formula for  $f^{-1}$ .

(8)

(ii) Show that  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  by using characteristic function. (8)

15. (a) (i) Show that the mapping  $g$  from the algebraic system  $(S, +)$  to the system  $(T, \cdot)$  defined by  $g(x) = 3x$ , where  $S$  is the set of all rational numbers under addition  $+$  and  $T$  is the set of non-zero real numbers under multiplication operation  $\cdot$ , is a homomorphism but not an isomorphism. (8)

(ii) Show that  $(2, 5)$  encoding function defined by  $e(00) = 00000$ ,  $e(01) = 01110$ ,  $e(10) = 10101$ ,  $e(11) = 11011$  is a group code. (8)

Or

Find the minimum distance of the encoding function  $e: \mathbb{Z}_2^2 \rightarrow \mathbb{B}^5$  given by  $e(00) = 00000$ ,  $e(01) = 01110$ ,  $e(10) = 10101$ ,  $e(11) = 11011$ . (8)

The intersection of any two subgroups of a group  $G$  is again a subgroup of  $G$ . - Prove. (8)

Let the function  $f$  and  $g$  be defined  $f(x) = 2x + 1$  and

Determine the composition function  $f \circ g$  and  $g \circ f$ .

Let  $a$  and  $b$  be positive integers and suppose

recursively as follows :

$f_0$ . if  $a < b$ )

th.b\=4

,

I

LQ(a-b,b)+1 if b<A)

Find Q(2,5 ), Q(I2,5),8(58617,)

g(r)= x ' - 2 .

(8)

A is defined

(8)

(i)

(ii)

(b)

c

,

1 1 0

0 0 1

0 0 1

???

?

?

???

?

?

= G A

???

?

?

???

?

?

=

1 0 0

1 0 0

0 1 1

H A . (8)

14. (a) (i) If  $(G, ?)$  is an abelian group, show that  $(a ? b) ? a = a ? (b ? a)$ . (8)

(ii) Show that  $(Z, +, \times)$  is an integral domain where  $Z$  is the set of all integers. (8)

Or

(b) (i) State and prove Lagrange's theorem. (8)

(ii) If  $(Z, +)$  and  $(E, +)$  where  $Z$  is the set all integers and  $E$  is the set all even integers, show that the two semi groups  $(Z, +)$  and  $(E, +)$  are isomorphic. (8)

15. (a) (i) Show that  $(N, ?)$  is a partially ordered set where  $N$  is set of all positive integers and  $?$  is defined by  $m ? n$  iff  $n - m$  is a non-negative integer. (8)

(ii) In a Boolean algebra, prove that  $(a ? b) ? = a ? ? b ?$ . (8)

Or



(b) (i) In a Lattice  $(L, \leq)$ , prove that  $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ . (8)

(ii) If  $S$  is the set all divisors of 42 and  $D$  is the relation “divisor of” on  $S$ , prove that  $(S, D)$  is a complemented Lattice. (8)