C 3282

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fifth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1256 — DISCRETE MATHEMATICS

(Common to B.E. (Part-Time) Fourth Semester Regulation 2005)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Express the statement "Good food is not cheap" in symbolic form.

2. Obtain PDNF for $\sim P \vee Q$.

3. Define simple statement function.

4. Express the statement "For every 'x' there exist a 'y' such that $x^2 + y^2 \geq 100$" in symbolic form.

5. Give an example of a relation which is symmetric, transitive but not reflexive on $\{a, b, c\}$.

6. Define partially ordered set.

7. If A has 3 elements and B has 2 elements, how many functions are there from A to B.

8. Define Characteristic function.

9. Give an example of sub semi-group.

10. Define normal subgroup of a group.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Without using truth tables, show that $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a tautology. (8)

(ii) Obtain the PDNF for $(P \wedge Q) \vee (\sim P \wedge Q) \vee (Q \wedge R)$. (8)

Or

(b) (i) Show that S is valid inference from the premises

$P \rightarrow \sim Q, Q \vee R, \sim S \rightarrow P$ and $\sim R$. (8)

(ii) Without using truth tables, show that

$(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (8)

12. (a) (i) Show that

$(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$ (8)

(ii) Is the following conclusion validly derivable from the premises given?

If $(\forall x)(P(x) \rightarrow Q(x))$, $(\exists y)P(y)$ then $(\exists z)Q(z)$ (8)

Or

(b) (i) Verify the validity of the inference. If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than Peter. Therefore, John is not successful than Peter. (8)

(ii) Show that $R \vee S$ is a valid conclusion from the premises

$C \vee D$, $(C \vee D) \rightarrow \sim H$, $\sim H \rightarrow (A \wedge \sim B)$ and $(A \wedge \sim B) \rightarrow (R \vee S)$ (8)

13. (a) (i) A survey of 500 television watchers produced the following information: 285 watch foot ball games; 195 watch hockey games: 115 watch Basket ball games: 45 watch foot ball and basket ball games: 70 watch foot ball and hockey games: 50 watch hockey and basket ball games: 50 do not watch any of the three games: How many people watch exactly one of the three games. (8)

(ii) In a Boolean algebra L prove that

$(a \wedge b)' = a' \vee b'$ for all $a, b \in L$. (8)

Or

(b) (i) Let the relation R be defined on the set of all real numbers by 'if x, y are real numbers, $x R y \Leftrightarrow x - y$ is a rational numbers'. Show that R is an equivalence relation. (8)

(ii) Draw the Hasse Diagram of the Lattice L of all subsets of $\{a, b, c\}$ under intersection and union. (8)

14. (a) (i) Let the function f and g be defined $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Determine the composition function $f \circ g$ and $g \circ f$. (8)

(ii) Let a and b be positive integers and suppose Q is defined recursively as follows :

$$Q(a,b) = \begin{cases} 0, & \text{if } a < b \\ Q(a-b,b)+1 & \text{if } b \le a \end{cases}$$

Find $Q(2,5)$, $Q(12,5)$, $Q(5861,7)$ (8)

Or

(b) (i) Let $f: R \to R$ be defined by $f(x) = 2x - 3$. Find a formula for f^{-1}. (8)

(ii) Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by using characteristic function. (8)

15. (a) (i) Show that the mapping g from the algebraic system $(S, +)$ to the system (T, X) defined by $g(a) = 3^a$, where S is the set all rational numbers under addition + and T is the set of non-zero real numbers under multiplication operation X, is a homomorphism but not an isomorphism. (8)

(ii) Show that (2, 5) encoding function defined by $e(00) = 00000$, $e(01) = 01110$, $e(10) = 10101$, $e(11) = 11011$ is a group code. (8)

Or

(b) (i) Find the minimum distance of the encoding function $e: B^2 \to B^4$ given by e(00) = 0000, e(10) = 0110, e(01) = 1011, e(11) = 1100. (8)

(ii) The intersection of any two subgroups of a group G is again a subgroup of G. – Prove. (8)
