T 8234

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Fifth Semester

Computer Science and Engineering

MA 1256 – DISCRETE MATHEMATICS

(Common to B.E. (Part–Time) Fourth Semester – Regulation 2005)

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If P, Q and R are statement variables, prove that

$$P \wedge ((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \Rightarrow R.$$

2. Prove that whenever $A \wedge B \Rightarrow C$, we also have $A \Rightarrow (B \rightarrow C)$ and vice versa.

3. Give an example to show that $(\exists x)(A(x) \wedge B(x))$ need not be a conclusion from $(\exists x)A(x)$ and $(\exists x)B(x)$.

4. Find the truth value of

$$(x)(P \rightarrow Q(x)) \vee (\exists x) R(x)$$

where

$$P : 2 > 1, \quad Q(x) : x > 3, \quad R(x) : x > 4,$$

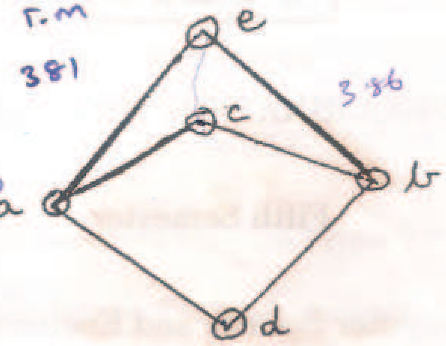
with the universe of discourse E being $E = \{2, 3, 4\}$.

5. For any sets A, B and C, prove that

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

$\{(x,y) \mid (x \in A) \wedge (y \in B \cap C)$

6. The following is the Hasse diagram of a partially ordered set. Verify whether it is a Lattice.



not lattices

Does not represent lattice because a ⊕ b not

7. If $f : A \to B$ and $g : B \to C$ are mappings and $gof : A \to C$ is one-to-one (Injection), prove that f is one-to-one.

8. If ψ_A denotes the characteristic function of the set A, prove that

$$\psi_{A \cup B}(x) = \psi_A(x) + \psi_B(x) - \psi_{A \cap B}(x),$$

Q.46 (Balaji)

for all $x \in E$, the universal set.

9. If S denotes the set of positive integers $\leq$ 100, for $x, y \in S$, define $x * y = \min\{x, y\}$. Verify whether $(S, *)$ is a Monoid assuming that $*$ is associative.

10. If H is a subgroup of the group G, among the right cosets of H in G, prove that there is only one subgroup viz., H.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that

$$(P \to Q) \wedge (Q \to R) \Rightarrow (P \to R). \quad (6)$$

(ii) Find the principal conjunctive and principal disjunctive normal forms of the formula

$$S \Leftrightarrow (P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R)). \quad (10)$$

Or

(b) (i) Using conditional proof prove that

$$\neg P \vee Q,\ \neg Q \vee R,\ R \to S \Rightarrow P \to S. \quad (8)$$

(ii) By using truth tables verify whether the following specifications are consistent : Whenever the system software is being upgraded users can not access the file system. If users can access the file system, then they can save new files. If users can not save new files then the system software is not being upgraded. (8)

12. (a) (i) Use indirect method of proof to show that 99 (T-m)

$$(x)\,(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)\,Q(x). \qquad (8)$$

(ii) Prove that

$$(\exists x)\,P(x) \to (x)\,Q(x) \Rightarrow (x)\,(P(x) \to Q(x)). \qquad (8)$$

Or

(b) (i) Use conditional proof to prove that

$$(x)(P(x) \to Q(x)) \Rightarrow (x)P(x) \to (x)\,Q(x). \qquad (8)$$

(ii) Prove that

$$(\exists)\,(A(x) \vee B(x)) \Leftrightarrow (\exists x)\,A(x) \vee (\exists x)\,B(x). \qquad (8)$$

13. (a) (i) Prove that distinct equivalence classes are disjoint. (4)

(ii) In a Lattice show that $a \le b$ and $c \le d$ implies $a * c \le b * d$. (4)

(iii) In a distributive lattice prove that $a * b = a * c$ and $a \oplus b = a \oplus c$ implies that $b = c$. (8)

Or

(b) (i) Let $\mathbf{P} = \{\{1, 2\}, \{3, 4\}, \{5\}\}$ be a partition of the set $S = \{1, 2, 3, 4, 5\}$. Construct an equivalence relation R on S so that the equivalence classes with respect to R are precisely the members of $\mathbf{P}$. (4)

(ii) Show that a chain with three or more elements is not complemented. Ex 103 (Bala) (4)

(iii) Establish De Morgan's laws in a Boolean Algebra. (8)

14. (a) (i) Find all mappings from $A = \{1, 2, 3\}$ to $B = \{4, 5\}$; find which of them are one-to one and which are onto. 4.7 Balaj. (8)

(ii) If

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix},$$

are permutations, prove that $(g \; o \; f)^{-1} = f^{-1} o g^{-1}$. (4)

(iii) If $\mathbf{R}$ denotes the set of real numbers and $f : \mathbf{R} \to \mathbf{R}$ is given by $f(x) = x^3 - 2$, find f^{-1}. 4-21 Ba (4)

Or

(b) (i) Let $\mathbf{Z}^{+}$ denote the set of positive integers and $\mathbf{Z}$ denote the set of integers. Let $f : \mathbf{Z}^{+} \to \mathbf{Z}$ be defined by

$$f(n) = \begin{cases} \dfrac{n}{2}, & \text{if } n \text{ is even} \\ \dfrac{1-n}{2}, & \text{if } n \text{ is odd} \end{cases} \qquad (8)$$

A.65

Prove that f is a bijection and find f^{-1}.

(ii) Let A, B, C be any three nonempty sets. Let $f : A \to B$ and $g : B \to C$ be mappings. If f and g are onto, prove that $g \circ f : A \to C$ is onto. Also give an example to show that $g \circ f$ may be onto but both f and g need not be onto. (8)

15. (a) (i) State and prove Lagrange's theorem for finite groups. (12)

(ii) Find all the non-trivial subgroups of $(\mathbf{Z}_6, +_6)$. (4)

Or

(b) If

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

is the Parity check matrix, find the Hamming code generated by H (in which the first three bits represent information portion and the next four bits are parity check bits). If $y = (0, 1, 1, 1, 1, 1, 0)$ is the received word find the corresponding transmitted code word. (16)